The Cosmic Microwave Background as a window on the Early Universe

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(accidental) discovery of the CMB

in the 60s, Penzias and Wilson were trying to remove all recognizable interference from their radio antenna, but were left with a residual **noise**.

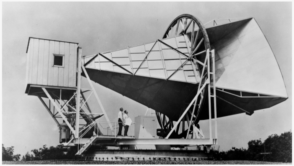

image credit: NASA

from *noise* to cosmological signature

our Universe at large scales

in our current understanding, our Universe can be described *on large scales* as being:

- homogeneous,
- isotropic,
- dynamic (expanding).

FRW metric: $$ds^2 = -dt^2 + a^2 \delta_{ij} dx^i dx^j$$

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FRW metric:

$$\begin{aligned} ds^2 &= -dt^2 + a^2 \delta_{ij} dx^i dx^j \\ &= a^2 \left(-d\eta^2 + \delta_{ij} dx^i dx^j\right) \end{aligned}$$

where a is the *scale factor* and $H \equiv \dot{a}/a$ is called *Hubble parameter*

cosmic dynamics

Einstein equations: $G_{\mu\nu} = 8\pi G\, T_{\mu\nu}.$

from FRW metric

assuming the Universe to be filled with a fluid ($T_{00} = -\rho$, $T_{ii} = p$)

Friedmann equations:

$$H^2 = \frac{8\pi G}{3}\rho\,,$$
$$2\dot{H} + 3H^2 = -8\pi G p\,.$$

Pressure

Radiation ($\gamma + \nu$): $p_r = \rho_r/3$.
Matter (ordinary + CDM): $p_m = 0$.
Dark Energy: $p_\Lambda = -\rho_\Lambda$.

Energy density

Radiation: $\rho_r \propto a^{-3}$.
Matter: $\rho_m \propto a^{-4}$.
Dark Energy: $\rho_\Lambda =$ const.

the behaviour of $a(t)$ can only be modeled if we know $\Omega_i = \rho_i/\rho_{\text{crit}}$.

a sprinkle of thermodynamics

decoupling of a species

as we go back in time, the Universe was denser and warmer: early enough, all species were in *thermal equilibrium* and their distribution function $f^{(0)}$ was determined by statistics only.

a species is said to *decouple* when all its interactions proceed slower than the Universe's expansion, i.e. when $\Gamma < H = \dot{a}/a$.

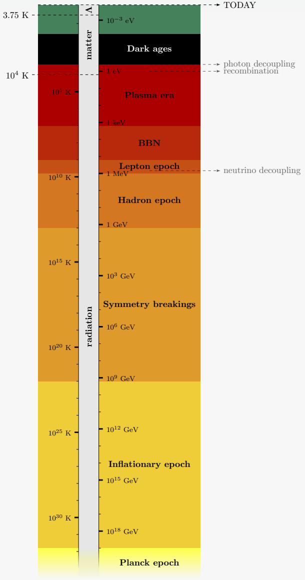
afterwards, $f = f^{(0)}$, but T behaves differently.

For photons:

$$f^{(0)} = \frac{1}{\exp(\nu/T) - 1},$$

with $T \propto a^{-1}$ before and after decoupling.

photon decoupling



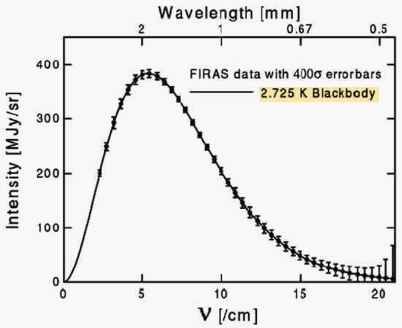
Recombination: as the Universe expanded, photons became less and less energetic, until they couldn't keep electrons and protons from combining into hydrogen atoms via $e^- + p \to \mathrm{H} + \gamma$.

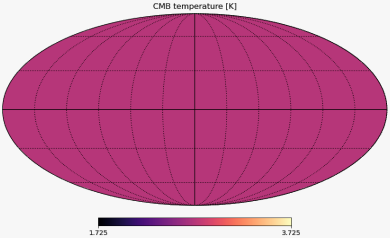
Photon decoupling: the drop of the number of free electrons, made it very unlikely for $e^- + \gamma \to e^- + \gamma$ scatterings to happen.

CMB: after decoupling, the photons travelled almost freely through space, characterized by a black-body spectrum.

observational evidence

the CMB appears to be a perfect **black-body** at $\overline{T} = 2.725$ K.





the temperature appears to be **isotropic** all over the sky.

good agreement between theoretical predictions for a FRW Universe and observations!

beyond isotropy

a more refined picture

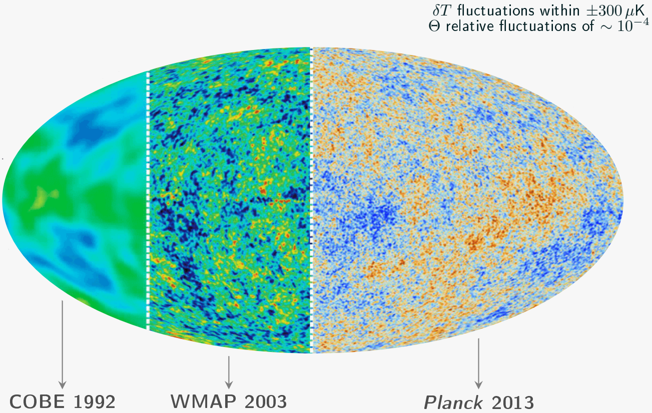


image credit: NASA

decomposition in spherical harmonics

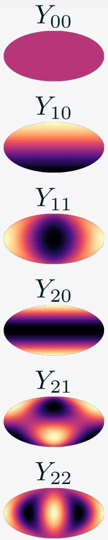
The **spherical harmonics** $Y_{\ell m}(\hat{\mathbf{n}})$ are eigenfunctions of the Laplace operator on the sphere.

They can be used to write $\Theta(\hat{\mathbf{n}}) = \sum_{\ell=0}^{\infty} \sum_{m=-\ell}^{\ell} a_{\ell m} Y_{\ell m}(\hat{\mathbf{n}})$,

with **coefficients** $a_{\ell m} \equiv \int \mathrm{d}^2 n \, \Theta(\hat{\mathbf{n}}) Y^*_{\ell m}(\hat{\mathbf{n}})$.

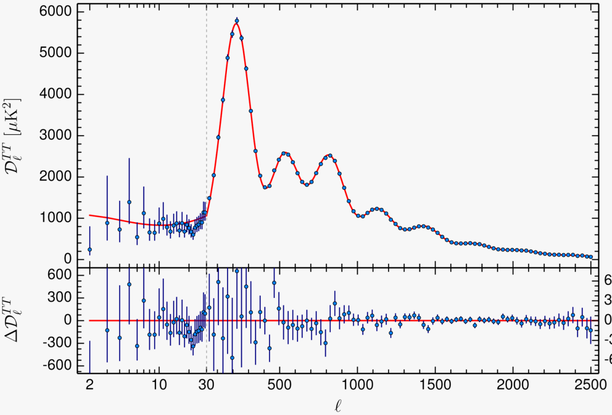
If T is an isotropic random field, $\langle a_{\ell m} a^*_{\ell' m'} \rangle = \delta_{\ell \ell'} \delta_{m m'} C_\ell$.

$$\boxed{C_\ell = \frac{1}{2\ell + 1} \sum_{m=-\ell}^{\ell} a_{\ell m} a^*_{\ell m}.}$$



Planck's angular power spectrum

$\mathcal{D}_\ell \equiv \ell(\ell+1)C_\ell/2\pi$



a lot of **physical information** is encoded in the power spectrum.

image credit: *Planck* collaboration

understanding anisotropies

perturbed metric tensor

in order to work with δT or, equivalently, $\Theta \equiv \delta T/\overline{T}$,
we need to abandon the isotropic and homogeneous description.

FRW metric: $ds^2 = -dt^2 + a^2\delta_{ij}dx^i dx^j$

perturbed: $ds^2 = -(1+2\Phi)dt^2 + a^2\left[(1-2\Psi)\delta_{ij} + h_{ij}\right]dx^i dx^j.$

cosmic fluids are also allowed to have perturbations: $\rho = \bar{\rho} + \delta\rho$.

geodesics in an expanding Universe

$$\frac{1}{p}\frac{dp}{dt} = -\frac{\dot{a}}{a} + \dot{\Psi} - \frac{1}{a}\sum_i \frac{\partial \Phi}{\partial x^i}\gamma^i - \frac{1}{2}\sum_{ij} \dot{h}_{ij}\gamma^i\gamma^j \,,$$

γ^i: unit vector in the direction of the photon momentum.

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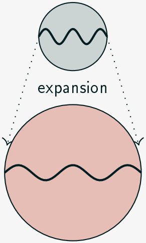
cosmological redshift: wavelengths stretch as $\lambda \propto a$ and the photons' energy drops: $p \propto a^{-1}$.



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$$ds^2 = a^2(t)(1-2\Psi)d\mathbf{x}^2 \equiv \tilde{a}^2(t,\mathbf{x})d\mathbf{x}^2,$$

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gravitational shift: additional gravitational red/blueshift due to tensor perturbations.

line of sight integration

integrating between the time of decoupling and today *(only scalar!)*:

$$\Theta(\hat{\mathbf{n}}) = \frac{\delta_\gamma(t_\mathsf{d}, \hat{\mathbf{n}} r_\mathsf{d})}{4} + \Phi(t_\mathsf{d}, \hat{\mathbf{n}} r_\mathsf{d}) - \Phi(t_0, \hat{\mathbf{n}} r_0) + \int_{t_\mathsf{d}}^{t_0} dt\, (\dot{\Phi} + \dot{\Psi})(t, \hat{\mathbf{n}} r).$$

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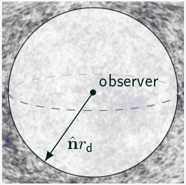
initial conditions: relative density fluctuations $\delta_\gamma(t_\mathsf{d}, \hat{\mathbf{n}} r_\mathsf{d}) \equiv \delta\rho_\gamma(t_\mathsf{d}, \hat{\mathbf{n}} r_\mathsf{d}) / \bar{\rho}_\gamma(t_\mathsf{d})$ at the time of photon decoupling.



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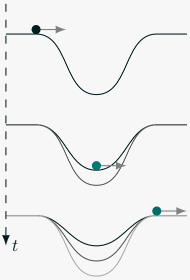
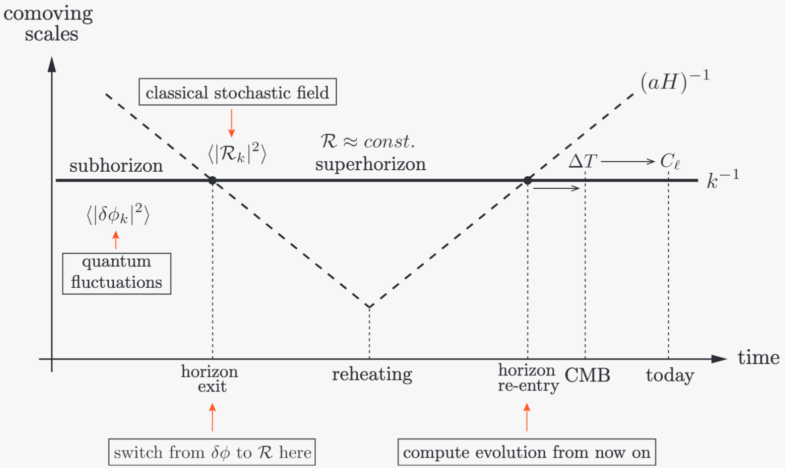
ISW effect: photons can gain more energy that they loose (or viceversa) if Φ depends on time.



initial conditions from inflation



a window on the Early Universe

$\Theta(\hat{\mathbf{n}})$ depends on the physics at decoupling via the initial conditions $[\delta_\gamma/4+\Phi](t_{\mathsf{d}},\hat{\mathbf{n}}r_{\mathsf{d}}) \equiv [\delta_\gamma/4+\Phi]_{\mathsf{d}}$, which can be related to the **primordial** Φ_{in} in Fourier space:

$$\left[\frac{\delta_\gamma}{4}+\Phi\right]_{\mathsf{d}} = \frac{3\Phi_{\mathsf{in}}}{10}\left[3R_b\mathcal{T}(k) - \frac{\mathcal{S}(k)\cos[kr_s+\Delta(k)]}{(1+R_b)^{1/4}}\right],$$

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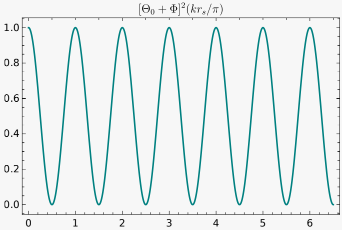

$$R_b = 0,$$
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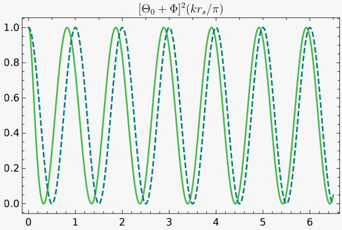

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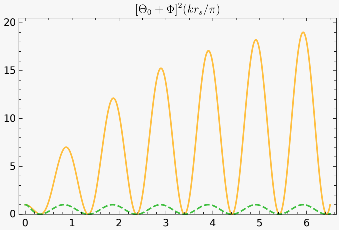

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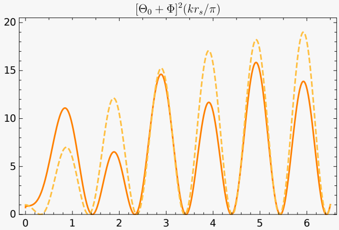

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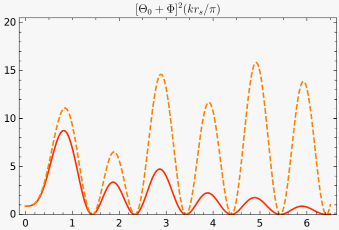

different photons decouple at slightly different times, leading to a suppression on small scales.

getting to the C_ℓ

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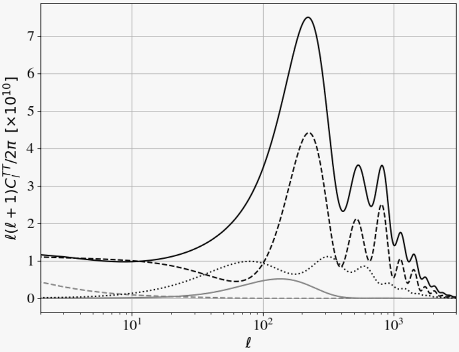
dashed black line

gray lines



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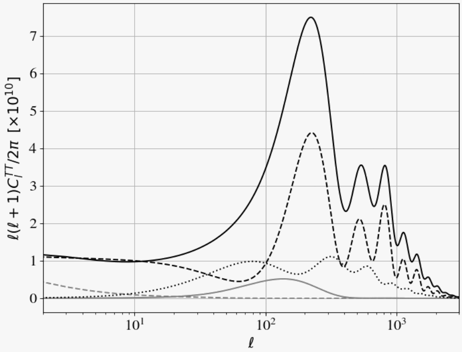
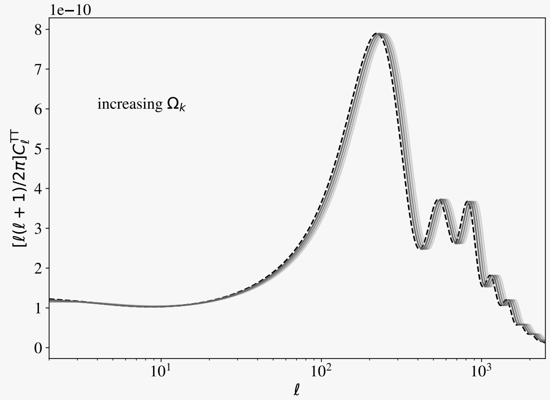


extracting cosmology from the C_ℓ



curvature k

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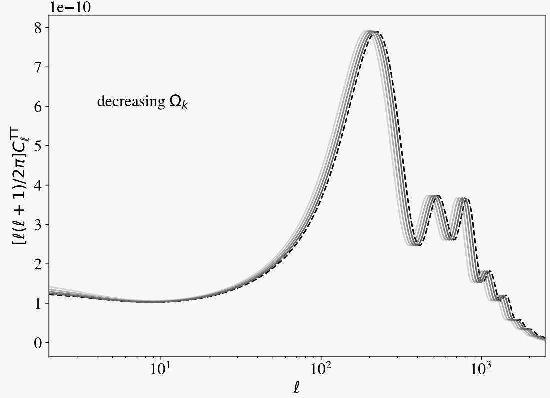

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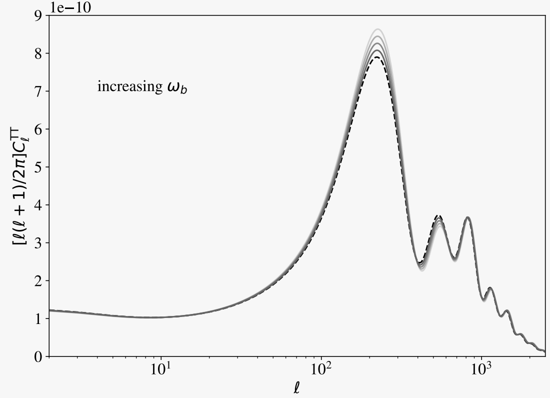

baryons

baryons shift vertically the oscillations at decoupling: when taking the square, even/odd peaks are enhanced/-suppressed.

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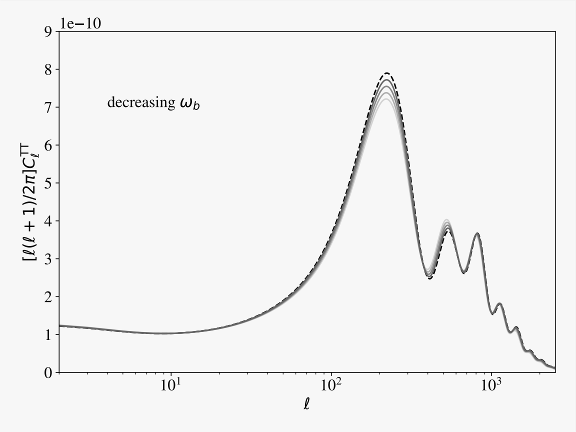

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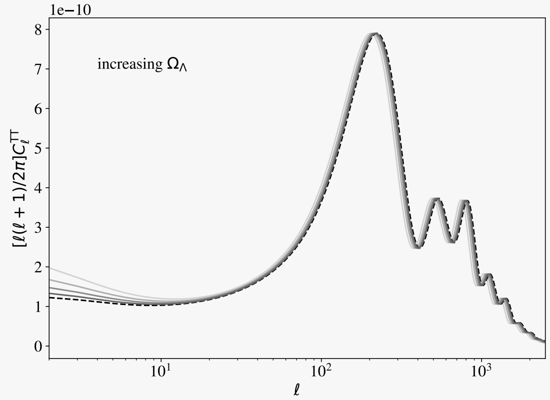

Dark Energy

Dark Energy cannot affect the physics at decoupling, but affects the ISW low-ℓ plateau.

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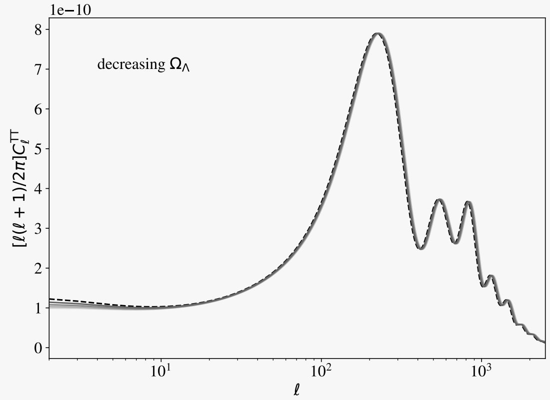

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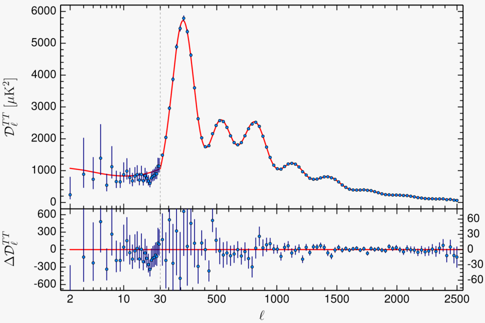
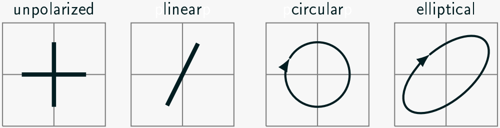
best-fit *Planck* cosmological parameters



Parameter	*Planck* alone
$\Omega_b h^2$	0.02237 ± 0.00015
$\Omega_c h^2$	0.1200 ± 0.0012
$100\theta_{MC}$	1.04092 ± 0.00031
τ	0.0544 ± 0.0073
$\ln(10^{10} A_s)$	3.044 ± 0.014
n_s	0.9649 ± 0.0042
H_0	67.36 ± 0.54
Ω_K	-0.0096 ± 0.0061
Σm_ν [eV]	< 0.241
N_{eff}	$2.89^{+0.36}_{-0.38}$
$r_{0.002}$	< 0.101

polarization

describing polarization



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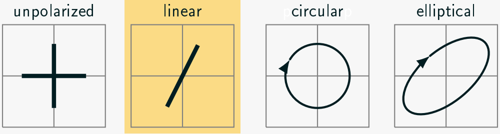

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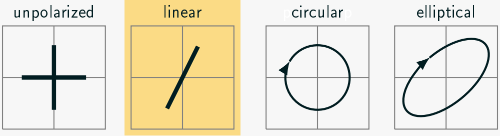
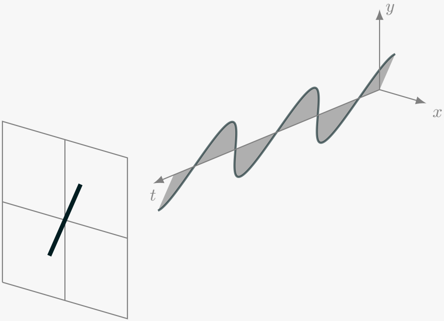
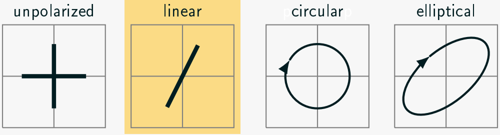
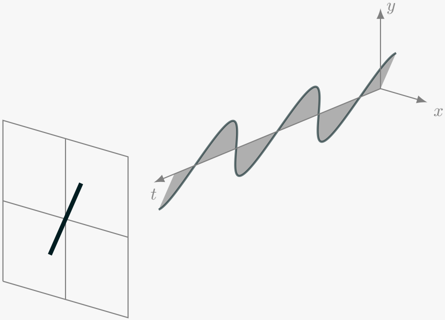
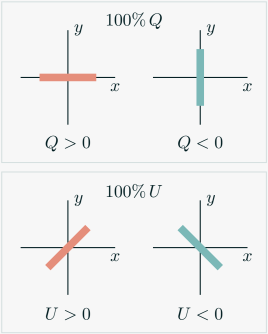




describing polarization







describing polarization: E- and B-modes

issue: (Q, U) are coordinate dependent!

for instance, rotating the coordinate system of $45°$ clockwise sends $Q \to -U$ and $U \to Q$.

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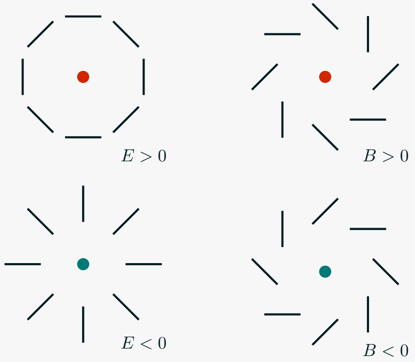
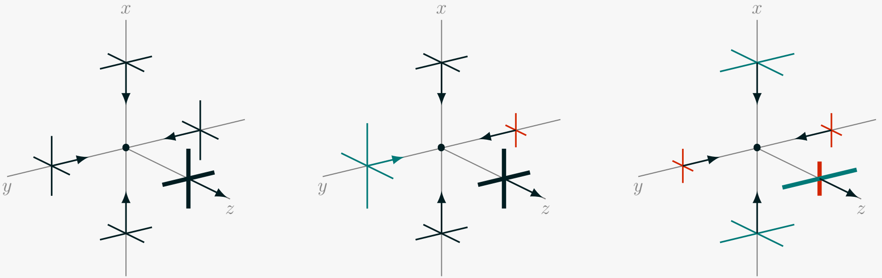
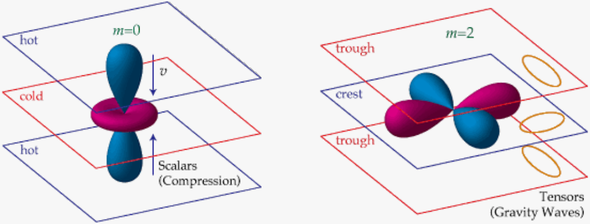


polarization from last-scattering



polarization can only be produced if the temperature distribution *around the electron at decoupling* has a quadrupole patter.

quadrupole from scalar vs tensor modes



both scalar and tensor modes can have a quadrupole pattern, however, **scalar modes can only generate E-modes**.

searching B-modes from inflation

Expectation: inflation-sourced perturbations leave traces on the CMB polarization.

Large scale B-modes can probe inflation.

Unprecedented sensitivity requirements!

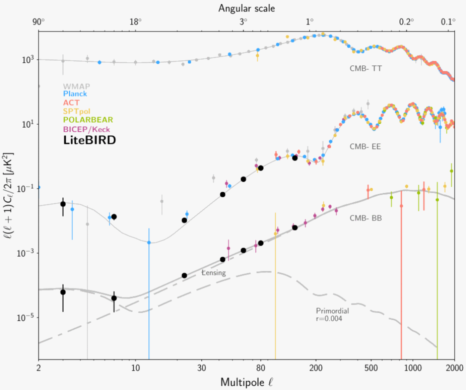
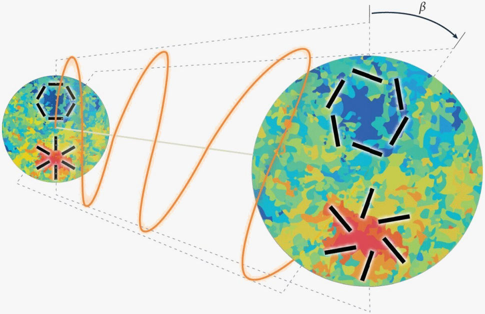


image credit: LiteBIRD Collaboration (2022) PTEP

a side effect: measuring cosmic birefringence

CMB might also carry information about
parity-violating new physics: **cosmic birefringence**.
(time-dependent parity-violating pseudoscalar field)

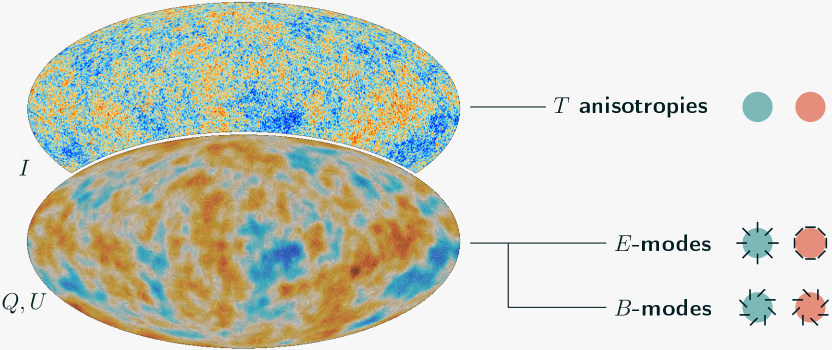


mixing of E and B modes: $\begin{cases} a^E_{\ell m,\mathsf{obs}} = a^E_{\ell m}\cos 2\beta - a^B_{\ell m}\sin 2\beta, \\ a^B_{\ell m,\mathsf{obs}} = a^E_{\ell m}\sin 2\beta + a^B_{\ell m}\cos 2\beta. \end{cases}$

image credit: Yuto Minami.

to sum up

CMB anisotropies



ongoing efforts to refine the detection of CMB polarization:
potential probe of inflation and cosmic birefringence.

image credit: Jonathan Aumont

backup

trying to constrain β

$$\begin{cases} C^{TT}_{\ell,\mathrm{obs}} = C^{TT}_\ell, \\ C^{EE}_{\ell,\mathrm{obs}} = \cos^2(2\beta)C^{EE}_\ell + \sin^2(2\beta)C^{BB}_\ell - \sin(4\beta)C^{EB}_\ell, \\ C^{BB}_{\ell,\mathrm{obs}} = \cos^2(2\beta)C^{BB}_\ell + \sin^2(2\beta)C^{EE}_\ell + \sin(4\beta)C^{EB}_\ell, \\ C^{TE}_{\ell,\mathrm{obs}} = \cos(2\beta)C^{TE}_\ell - \sin(2\beta)C^{TB}_\ell, \\ C^{EB}_{\ell,\mathrm{obs}} = \sin(4\beta)(C^{EE}_\ell - C^{BB}_\ell)/2 + \cos(4\beta)C^{EB}_\ell, \\ C^{TB}_{\ell,\mathrm{obs}} = \sin(2\beta)C^{TE}_\ell + \cos(2\beta)C^{TB}_\ell. \end{cases}$$

$$C^{EB}_{\ell,\mathrm{obs}} = \tan(4\beta)(C^{EE}_{\ell,\mathrm{obs}} - C^{BB}_{\ell,\mathrm{obs}})/2.$$

trying to constrain β

$$
\begin{cases}
C_{\ell,\mathrm{obs}}^{TT} = C_\ell^{TT}, \\
C_{\ell,\mathrm{obs}}^{EE} = \cos^2(2\beta)C_\ell^{EE} + \sin^2(2\beta)C_\ell^{BB} - \sin(4\beta)C_\ell^{EB}, \\
C_{\ell,\mathrm{obs}}^{BB} = \cos^2(2\beta)C_\ell^{BB} + \sin^2(2\beta)C_\ell^{EE} + \sin(4\beta)C_\ell^{EB}, \\
C_{\ell,\mathrm{obs}}^{TE} = \cos(2\beta)C_\ell^{TE} - \sin(2\beta)C_\ell^{TB}, \\
C_{\ell,\mathrm{obs}}^{EB} = \sin(4\beta)(C_\ell^{EE} - C_\ell^{BB})/2 + \cos(4\beta)C_\ell^{EB}, \\
C_{\ell,\mathrm{obs}}^{TB} = \sin(2\beta)C_\ell^{TE} + \cos(2\beta)C_\ell^{TB}.
\end{cases}
$$

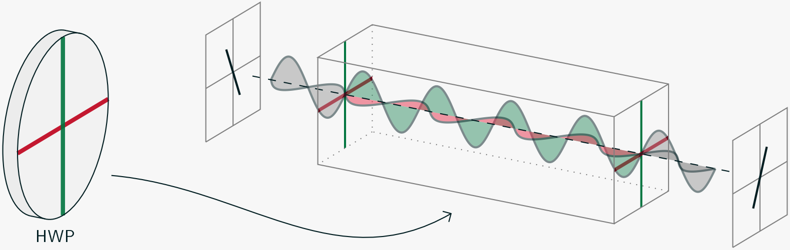
$$
\longrightarrow \quad C_{\ell,\mathrm{obs}}^{EB} = \tan(4\beta)(C_{\ell,\mathrm{obs}}^{EE} - C_{\ell,\mathrm{obs}}^{BB})/2.
$$

$\beta = 0.35 \pm 0.14$ (68%CL)

Minami and Komatsu (2020) Phys. Rev. Lett. 125

To extract this kind of information from CMB systematics have to be kept under control.

the HWP: reducing systematics



A **rotating** half-wave plate (HWP) as first optical element:

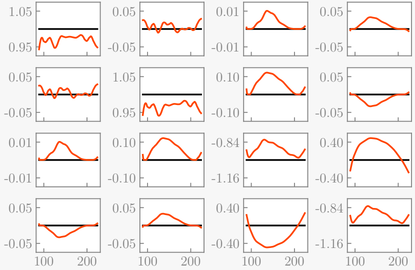
- modulates the signal to $4f_{\mathrm{HWP}}$, allowing to “escape” $1/f$ noise;
- makes possible for a single detector to measure polarization, reducing pair-differencing systematics.

the HWP: inducing systematics

Mueller calculus: radiation described as $S = (I, Q, U, V)$, effect of polarization-altering devices parametrized by $\mathcal{M}$: so that $\mathbf{S}' = \mathcal{M} \cdot \mathbf{S}$.

For an ideal HWP, $\mathcal{M}_{\text{ideal}} = \text{diag}(1, 1, -1, -1)$, but let's look at a realistic case:

$\mathcal{M}_{\text{HWP}} = \Big($



$\Big)\,.$

how does this affect the observed maps?

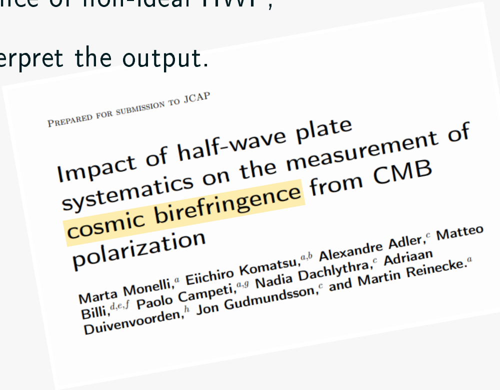
Giardiello et al. (2022) A&A 658

steps we took in that direction

work on a simulation pipeline for a LiteBIRD-like mission;

simulate observed maps in presence of non-ideal HWP;

derive analytical formulae to interpret the output.


Prepared for submission to JCAP

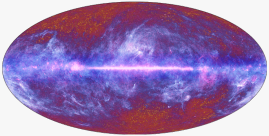
Impact of half-wave plate systematics on the measurement of cosmic birefringence from CMB polarization

Marta Monelli,[a] Eiichiro Komatsu,[a,b] Alexandre Adler,[c] Matteo Billi,[d,e,f] Paolo Campeti,[a,g] Nadia Dachlythra,[c] Adriaan Duivenvoorden,[h] Jon Gudmundsson,[c] and Martin Reinecke.[a]

Monelli et al. submitted to JCAP, eprint: `arXiv:2211.05685`

simulations

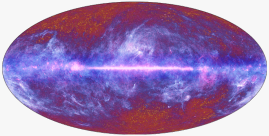
what do we simulate



CMB sky maps

image credit: *Planck* collaboration

what do we simulate



detection

time ordered data (TOD)

map-making

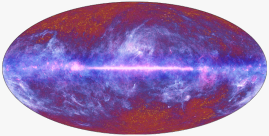
multi-frequency maps

foreground cleaning

CMB sky maps

image credit: *Planck* collaboration

what do we simulate



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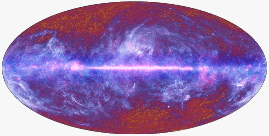
CMB sky maps

TOD: collection of the signal detected by *each of the (4508) detectors* during *the whole (3-year) mission*.

image credit: *Planck* collaboration

what do we simulate



detection

time ordered data (TOD)

map-making

multi-frequency maps

foreground cleaning

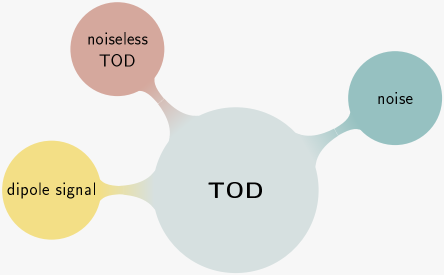
CMB sky maps

TOD: collection of the signal detected by *each of the (4508) detectors* during *the whole (3-year) mission*.

Simulating TOD is crucial in the planning of any CMB experiment: helps studying potential systematic effects.

image credit: *Planck* collaboration

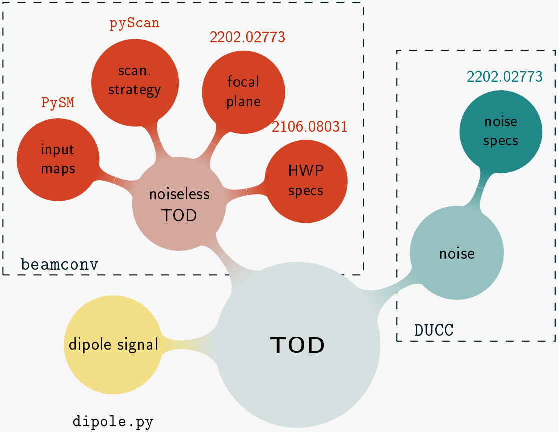
sketch of the pipeline



sketch of the pipeline

beamconv: convolution code simulating TOD for CMB experiments with realistic polarized **beams**, scanning strategies and **HWP**.

DUCC: collection of basic programming tools for numerical computation: `fft`, `sht`, `healpix`, `totalconvolve`...



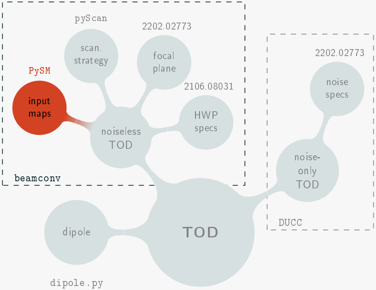
github.com/AdriJD/beamconv, A. Duivenvoorden et al "2012.10437", gitlab.mpcdf.mpg.de/mtr/ducc

working assumptions

To focus on the impact of **HWP non-idealities**, we consider a simplified problem:

- no noise,
- single frequency,
- CMB-only,
- simple beams,
- HWP aligned to the detector line of sight.

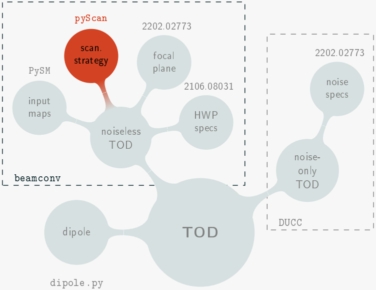
input maps



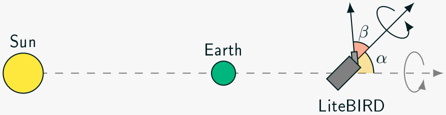
The pipeline can be fed with arbitrary input maps: CMB, foregrounds, or both.

In the paper: I, Q and U input maps with $n_{\text{side}} = 512$ from best-fit 2018 Planck power spectra;

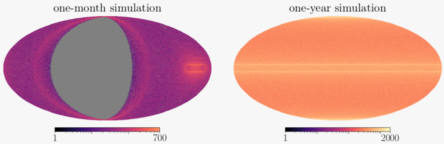
scanning strategy



The pipeline can read or calculate pointings. We implemented some functionalities of pyScan in beamconv to deal with satellite missions.

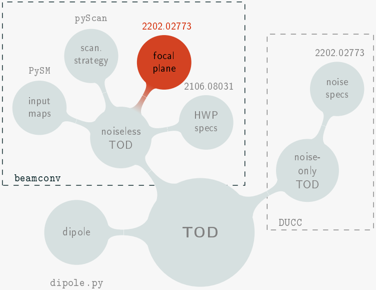


In the paper: 1 year of LiteBIRD-like scanning strategy.



https://github.com/tmatsumu/LB_SYSPL_v4.2

focal plane specifics



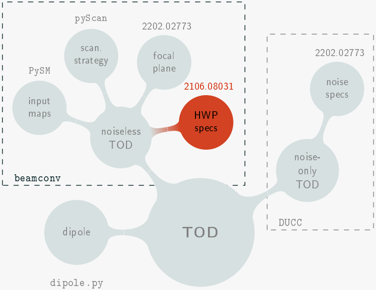
The pipeline can read from the Instrument Model Database (IMO):

```
{'name': 'M02_030_QA_140T',
 'wafer': 'M02',
 'pixel': 30,
 'pixtype': 'MP1',
 [...]
 'pol': 'T',
 'orient': 'Q',
 'quat': [1, 0, 0, 0]}
```

In the paper: 160 dets from M1-140.

specs.	values
f_{samp}	19 Hz
HWP rpm	39
FWHM	30.8 arcmin
offset quats.	[...]

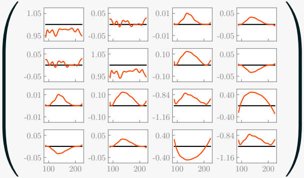
HWP specifics



In the paper: HWP is assumed to be ideal in the **first** simulation run (ideal TOD) and realistic in the **second** (non-ideal TOD).

Realistic HWP Mueller matrix elements as shown previously:

$$\mathcal{M}_{\mathrm{HWP}} =$$

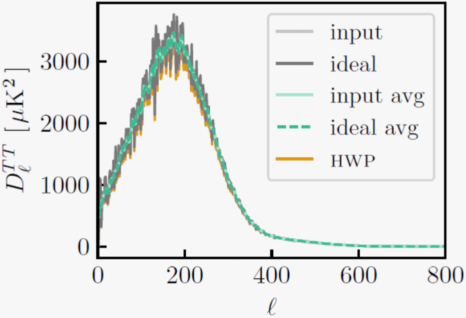


requency [GHz]

what about maps?

Both ideal and non-ideal TOD processed by **ideal** bin-averaging map-maker.

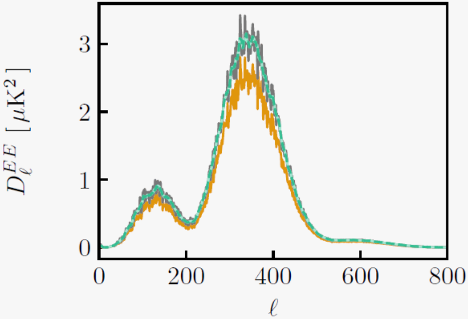
ideal vs non-ideal output spectra (1)



(beam transfer function not deconvolved)

TT leaked a bit

ideal vs non-ideal output spectra (1)

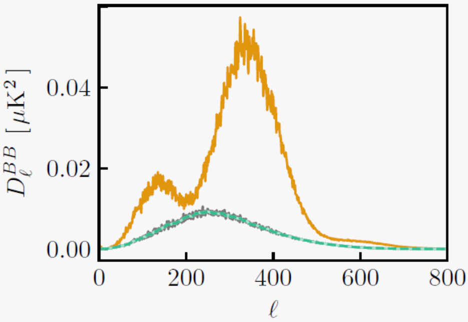


(beam transfer function not deconvolved)

TT leaked a bit

EE leaked a lot!

ideal vs non-ideal output spectra (1)



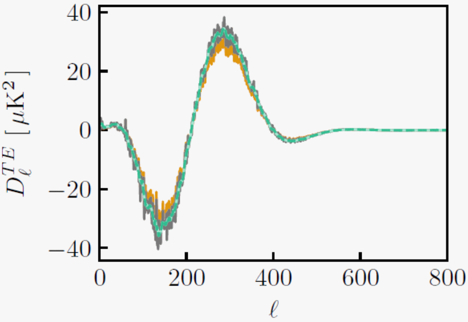
(beam transfer function not deconvolved)

TT leaked a bit

EE leaked a lot!

BB larger (EE shape!)

ideal vs non-ideal output spectra (1)



(beam transfer function not deconvolved)

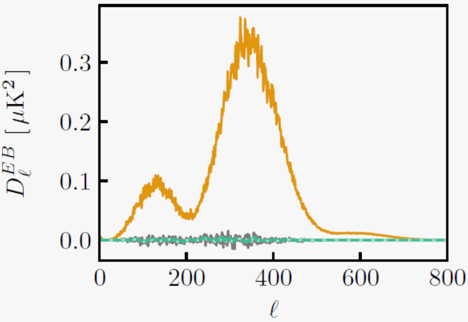
TT leaked a bit

EE leaked a lot!

BB larger (EE shape!)

TE leaked a bit

ideal vs non-ideal output spectra (1)



(beam transfer function not deconvolved)

TT leaked a bit

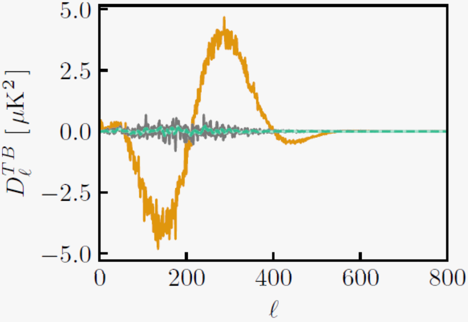
EE leaked a lot!

BB larger (EE shape!)

TE leaked a bit

EB non-zero!

ideal vs non-ideal output spectra (1)



(beam transfer function not deconvolved)

TT leaked a bit

EE leaked a lot!

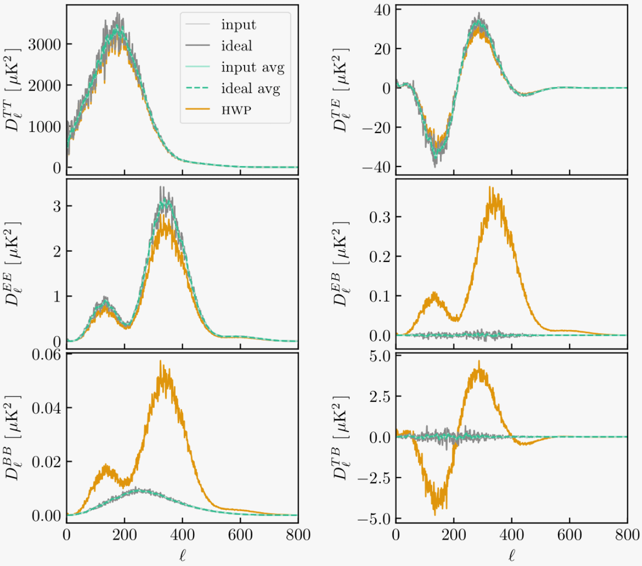
BB larger (EE shape!)

TE leaked a bit

EB non-zero!

TB non-zero!

ideal vs non-ideal output spectra (2)



how can we understand this?

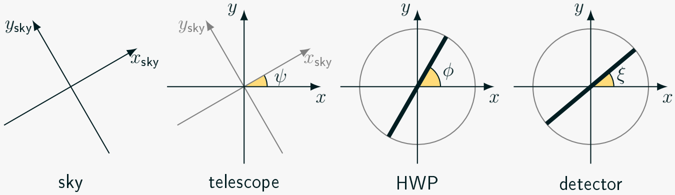
modeling the TOD

How `beamconv` computes the TOD:

$$d_t = \sum_{s\ell m} \left[B^I_{\ell s}\, a^I_{\ell m} + \frac{1}{2} \left({}_{-2}B^P_{\ell s}\, {}_2a^P_{\ell m} + {}_2B^P_{\ell s}\, {}_{-2}a^P_{\ell m} \right) + B^V_{\ell s}\, a^V_{\ell m} \right] \sqrt{\frac{4\pi}{2\ell+1}} e^{-is\psi_t}\, {}_sY_{\ell m}(\theta_t, \phi_t)\,,$$

beam coefficients *(or combinations of them if HWP non-ideal)*.

In the paper: $d = (1\ 0\ 0) \cdot \mathcal{M}_{\mathsf{det}} \mathcal{R}_{\xi-\phi} \mathcal{M}_{\mathsf{HWP}} \mathcal{R}_{\phi+\psi} \cdot \mathbf{S}$.



modeling the observed maps

(minimal) TOD: signal detected by 4 detectors.
map-maker: bin-averaging assuming ideal HWP.
estimated output maps: linear combination of $\{I, Q, U\}_{\text{in}}$.

$$\begin{pmatrix} d^{(0)} \\ d^{(90)} \\ d^{(45)} \\ d^{(135)} \end{pmatrix} = \begin{pmatrix} (1\ 0\ 0) \cdot \mathcal{M}_{\text{det}} \mathcal{R}_{0-\phi} \mathcal{M}_{\text{HWP}} \mathcal{R}_{\phi+\psi} \\ (1\ 0\ 0) \cdot \mathcal{M}_{\text{det}} \mathcal{R}_{90-\phi} \mathcal{M}_{\text{HWP}} \mathcal{R}_{\phi+\psi} \\ (1\ 0\ 0) \cdot \mathcal{M}_{\text{det}} \mathcal{R}_{45-\phi} \mathcal{M}_{\text{HWP}} \mathcal{R}_{\phi+\psi} \\ (1\ 0\ 0) \cdot \mathcal{M}_{\text{det}} \mathcal{R}_{135-\phi} \mathcal{M}_{\text{HWP}} \mathcal{R}_{\phi+\psi} \end{pmatrix} \cdot \begin{pmatrix} I_{\text{in}} \\ Q_{\text{in}} \\ U_{\text{in}} \end{pmatrix}$$

Being *ideal*, map-making amounts to apply $(\widehat{A}^T \widehat{A})^{-1} \widehat{A}^T$ to the TOD:

$$\widehat{S} = (\widehat{A}^T \widehat{A})^{-1} \widehat{A}^T A \cdot S.$$

estimated ouput maps

$$
\begin{aligned}
\widehat{I} &= m_{ii} I_{\mathsf{in}} + (m_{iq} Q_{\mathsf{in}} + m_{iu} U_{\mathsf{in}}) \cos(2\alpha) + (m_{iq} U_{\mathsf{in}} - m_{iu} Q_{\mathsf{in}}) \sin(2\alpha)\,, \\
\widehat{Q} &= \frac{1}{2}\Big\{ (m_{qq} - m_{uu}) Q_{\mathsf{in}} + (m_{qu} + m_{uq}) U_{\mathsf{in}} + 2 m_{qi} I_{\mathsf{in}} \cos(2\alpha) + 2 m_{ui} I_{\mathsf{in}} \sin(2\alpha) \\
&\qquad + \big[(m_{qq} + m_{uu}) Q_{\mathsf{in}} + (m_{qu} - m_{uq}) U_{\mathsf{in}} \big] \cos(4\alpha) \\
&\qquad + \big[-(m_{qu} - m_{uq}) Q_{\mathsf{in}} + (m_{qq} + m_{uu}) U_{\mathsf{in}} \big] \sin(4\alpha) \Big\}\,, \\
\widehat{U} &= \frac{1}{2}\Big\{ (m_{qq} - m_{uu}) U_{\mathsf{in}} - (m_{qu} + m_{uq}) Q_{\mathsf{in}} - 2 m_{ui} I_{\mathsf{in}} \cos(2\alpha) + 2 m_{qi} I_{\mathsf{in}} \sin(2\alpha) \\
&\qquad + \big[-(m_{qq} + m_{uu}) U_{\mathsf{in}} + (m_{qu} - m_{uq}) Q_{\mathsf{in}} \big] \cos(4\alpha) \\
&\qquad + \big[(m_{qu} - m_{uq}) U_{\mathsf{in}} + (m_{qq} + m_{uu}) Q_{\mathsf{in}} \big] \sin(4\alpha) \Big\}\,,
\end{aligned}
$$

where $\alpha = \phi + \psi$. For **good** coverage and **rapidly spinning** HWP:

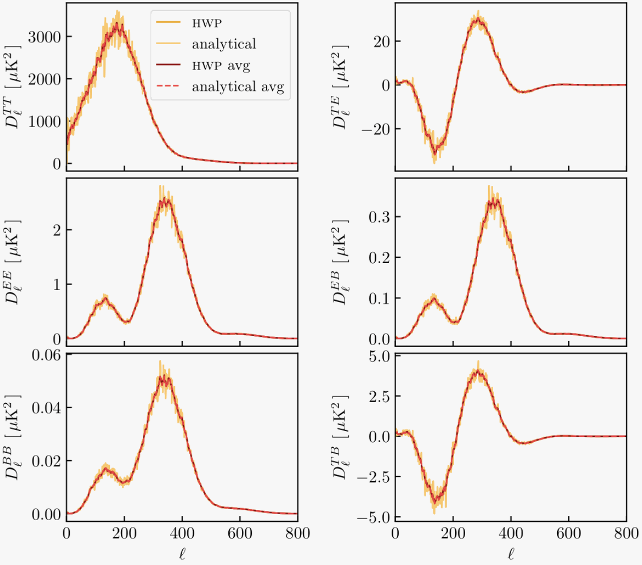
$$
\widehat{\mathbf{S}} \simeq \begin{pmatrix} m_{ii} I_{\mathsf{in}} \\ [(m_{qq} - m_{uu}) Q_{\mathsf{in}} + (m_{qu} + m_{uq}) U_{\mathsf{in}}]/2 \\ [-(m_{qu} + m_{uq}) Q_{\mathsf{in}} + (m_{qq} - m_{uu}) U_{\mathsf{in}}]/2 \end{pmatrix}.
$$

equations for the $\widehat{C}_\ell$s

Expanding $\widehat{S}$ in spherical harmonics:

$$
\begin{aligned}
\widehat{C}_\ell^{TT} &\simeq m_{ii}^2 C_{\ell,\mathsf{in}}^{TT}, \\
\widehat{C}_\ell^{EE} &\simeq \frac{(m_{qq}-m_{uu})^2}{4} C_{\ell,\mathsf{in}}^{EE} + \frac{(m_{qu}+m_{uq})^2}{4} C_{\ell,\mathsf{in}}^{BB} + \frac{(m_{qq}-m_{uu})(m_{qu}+m_{uq})}{2} C_{\ell,\mathsf{in}}^{EB}, \\
\widehat{C}_\ell^{BB} &\simeq \frac{(m_{qq}-m_{uu})^2}{4} C_{\ell,\mathsf{in}}^{BB} + \frac{(m_{qu}+m_{uq})^2}{4} C_{\ell,\mathsf{in}}^{EE} - \frac{(m_{qq}-m_{uu})(m_{qu}+m_{uq})}{2} C_{\ell,\mathsf{in}}^{EB}, \\
\widehat{C}_\ell^{TE} &\simeq \frac{m_{ii}(m_{qq}-m_{uu})}{2} C_{\ell,\mathsf{in}}^{TE} + \frac{m_{ii}(m_{qu}+m_{uq})}{2} C_{\ell,\mathsf{in}}^{TB}, \\
\widehat{C}_\ell^{EB} &\simeq \frac{(m_{qq}-m_{uu})^2-(m_{qu}+m_{uq})^2}{4} C_{\ell,\mathsf{in}}^{EB} - \frac{(m_{qq}-m_{uu})(m_{qu}+m_{uq})}{2} (C_{\ell,\mathsf{in}}^{EE} - C_{\ell,\mathsf{in}}^{BB}), \\
\widehat{C}_\ell^{TB} &\simeq \frac{m_{ii}(m_{qq}-m_{uu})}{2} C_{\ell,\mathsf{in}}^{TB} - \frac{m_{ii}(m_{qu}+m_{uq})}{2} C_{\ell,\mathsf{in}}^{TE}.
\end{aligned}
$$

analytical vs non-ideal output spectra



impact on cosmic birefringence

HWP-induced miscalibration

Analytic $\widehat{C}_\ell$s satisfy the relations:

$$\begin{cases} \widehat{C}_\ell^{EB} \simeq \tan(4\widehat{\theta}) \left[\widehat{C}_\ell^{EE} - \widehat{C}_\ell^{BB}\right]/2 \\ \widehat{C}_\ell^{TB} \simeq \tan(2\widehat{\theta}) \widehat{C}_\ell^{TE} \end{cases}$$

The HWP induces an additional miscalibration, **degenerate** with cosmic birefringence and polarization angle miscalibration!

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our formulae suggest

$$\widehat{\theta} \equiv -\frac{1}{2} \arctan \frac{m_{qu} + m_{uq}}{m_{qq} - m_{uu}} \sim 3.8^\circ,$$

compatibly with simulations.

The HWP induces an additional miscalibration, **degenerate** with cosmic birefringence and polarization angle miscalibration!

This doesn't mean that the HWP will keep us from measuring β, but it shows how important it is to carefully calibrate $\mathcal{M}_{\text{HWP}}$.

simple generalizations

including frequency dependence

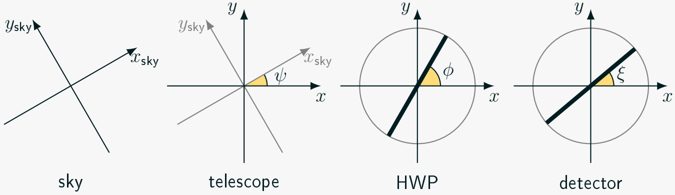
How does $d = (1\ 0\ 0) \cdot \mathcal{M}_{\mathsf{det}} \mathcal{R}_{\xi-\phi} \mathcal{M}_{\mathsf{HWP}} \mathcal{R}_{\phi+\psi} \cdot \mathbf{S}$ change when the **frequency dependence** of $\mathcal{M}_{\mathsf{HWP}}$ and signal is taken into account?

$$d = (1\ 0\ 0) \cdot \mathcal{M}_{\mathsf{det}} \mathcal{R}_{\xi-\phi} \int \mathsf{d}\nu\, \mathcal{M}_{\mathsf{HWP}}(\nu) \mathcal{R}_{\phi+\psi} \cdot \mathbf{S}(\nu)\,.$$

Assuming an ideal map-maker and retracing the same steps as before:

$$\widehat{\theta} = -\frac{1}{2} \arctan \frac{\langle m_{qu} + m_{uq} \rangle}{\langle m_{qq} - m_{uu} \rangle}, \qquad \text{where } \langle \cdot \rangle = \int \mathsf{d}\nu \cdot (\nu) S(\nu).$$

instrument miscalibration



So far, we assumed $\begin{cases} \widehat{\psi} \equiv \psi, \\ \widehat{\phi} \equiv \phi, \\ \widehat{\xi} \equiv \xi, \end{cases}$ but more generally $\begin{cases} \widehat{\psi} \equiv \psi + \delta\phi, \\ \widehat{\phi} \equiv \phi + \delta\psi, \\ \widehat{\xi} \equiv \xi + \delta\xi. \end{cases}$

Taking such (frequency-independent) deviations into account:

$$\widehat{\theta} = -\frac{1}{2} \arctan \frac{\langle m_{qu} + m_{uq} \rangle}{\langle m_{qq} - m_{uu} \rangle} + \delta\theta, \qquad \text{where } \delta\theta \equiv \delta\xi - \delta\psi - 2\delta\phi.$$

steps forward

Even *more general* generalizations worth exploring:

- including a realistic band pass,
- allowing for miscalibrations to depend on ν.

For how long can we push the analytical formulae?

the importance of calibration

how does the map-model change

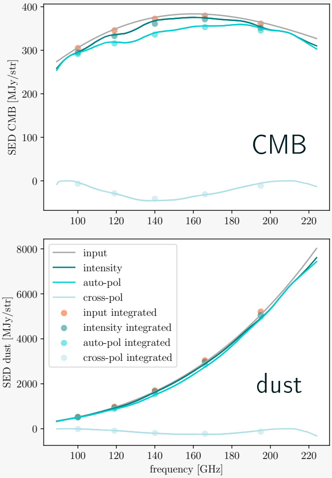
Without HWP:
$$\begin{pmatrix} I_j \\ Q_j \\ U_j \end{pmatrix} = \sum_\lambda g_\lambda \begin{pmatrix} I_\lambda \\ Q_\lambda \\ U_\lambda \end{pmatrix} + n,$$

With HWP:
$$\begin{pmatrix} I_j \\ Q_j \\ U_j \end{pmatrix} = \sum_\lambda \begin{pmatrix} g_\lambda^{ii} & 0 & 0 \\ 0 & g_\lambda^{qq-uu} & g_\lambda^{qu+uq} \\ 0 & g_\lambda^{qu+uq} & g_\lambda^{qq-uu} \end{pmatrix} \begin{pmatrix} I_\lambda \\ Q_\lambda \\ U_\lambda \end{pmatrix} + n,$$

where $g_\lambda = \frac{\int \mathrm{d}\nu\, G(\nu) S_\lambda(\nu)}{\int \mathrm{d}\nu\, G(\nu)}$, $g_\lambda^{ii} = \frac{\int \mathrm{d}\nu\, G(\nu) m_{ii}(\nu) S_\lambda(\nu)}{\int \mathrm{d}\nu\, G(\nu)}$, and so on.

HWP non-idealities contribute to gain, polarization-efficiency and cross-polarization leakage.

effective SEDs



$$\sum_{\lambda} \begin{pmatrix} g_{\lambda}^{ii} & 0 & 0 \\ 0 & g_{\lambda}^{qq-uu} & g_{\lambda}^{qu+uq} \\ 0 & g_{\lambda}^{qu+uq} & g_{\lambda}^{qq-uu} \end{pmatrix} \begin{pmatrix} I_{\lambda} \\ Q_{\lambda} \\ U_{\lambda} \end{pmatrix} + n,$$

Since all these effects are frequency dependent, they affect each component differently,

An imprecise calibration of $\mathcal{M}_{\mathrm{HWP}}$ can lead to complications in the component separation step.

- we are now provided with a simulation pipeline that can be easily adapted to study more complex problems (adding noise, more realistic beams...);
- the analytical formulae represent an alternative tool to study the same problems more effectively (but approximately);
- obvious application: exploiting the analytical formulae to derive calibration requirements for the HWP Mueller matrix elements, so that they don't prevent us from detecting B-modes, measuring cosmic birefringence, nor spoil the foreground cleaning procedure.