

#### Impact of half-wave plate systematics on the observed CMB polarization

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CMB Group Meeting, KEK, June 21st, 2023

Inhomogeneities at photon decoupling imprint anisotropies on the CMB.



Inhomogeneities at photon decoupling imprint anisotropies on the CMB.



 $\triangleright$  Inflation-sourced tensor perturbations are expected to leave a distinctive signature (B-modes) on CMB polarization:

$$
C_{\ell}^{BB} = rC_{\ell}^{GW} + C_{\ell}^{lensing}.
$$

This is driving the development of a number of new missions:

- Simons Observatory,
- South Pole Observatory,
- $\Box$  CMB Stage-4,

LiteBIRD.



# <span id="page-4-0"></span>[measuring polarization](#page-4-0)

### describing polarization: Stokes vectors



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#### describing polarization: Stokes vectors



**Mueller calculus:** radiation described as  $S = (I, Q, U)$ , effect of polarization-altering devices parametrized by M so that  $S' = \mathcal{M} \cdot S$ .

$$
\mathcal{M}_{\mathsf{pol}} = \frac{1}{2} \begin{pmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \mathcal{M}_{\theta} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos(2\theta) & \sin(2\theta) \\ 0 & -\sin(2\theta) & \cos(2\theta) \end{pmatrix}, \quad \ldots
$$

Given two optical elements in series with  $\mathcal{M}_1$  and  $\mathcal{M}_2$ , their combined effect can be described by  $M_2M_1$ .

Matrix methods are extremely convenient to manipulate polarization, since one does not work with the electromagnetic field itself.



This method can lead to detection of spurious polarization.

#### How will next generation CMB experiments deal with this?

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- $\Box$  Simons Observatory,
- $\Box$  South Pole Observatory,
- $\Box$  CMB Stage-4.

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- South Pole Observatory,
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They all plan to employ rotating half-wave plates (HWPs) as polarization modulators.

### the HWP: reducing systematics



A rotating half-wave plate (HWP) as first optical element can help to control systematics.

























incoming

 $\triangleright$  The intrinsic signal is modulated to  $4f_{\text{HWP}}$  and can be distinguished from spurious signal (no/different modulation).

### the HWP Mueller matrix

For an ideal HWP,  $\mathcal{M}_{HWP}$  is simply  $\mathcal{M}_{ideal} = diag(1, 1, -1)$ , but things get more complicated for realistic cases:



How does this affect the observed maps?

# <span id="page-25-0"></span>[simulating/modeling the HWP effect](#page-25-0)

#### how to propagate systematics



#### how to propagate systematics



TOD: collection of the signal detected by each of the (4508) detectors during the whole (3-year) mission.

image credit: Planck collaboration

#### how to propagate systematics



TOD: collection of the signal detected by each of the (4508) detectors during the whole (3-year) mission.

Simulating and modeling TOD is crucial in the planning of any CMB experiment: it helps studying potential systematic effects.

Realistic simulations are key for the study of systematics, because they can account for them in their (at least partial) complexity.

Approximate models, on the other hand, are extremely useful to gain some *intuition* about the problem at hand.

Realistic simulations are key for the study of systematics, because they can account for them in their (at least partial) complexity.

Approximate models, on the other hand, are extremely useful to gain some intuition about the problem at hand.

Just in case, we did both.

#### beamconv-based simulation pipeline



[github.com/AdriJD/beamconv,](https://github.com/AdriJD/beamconv) [A. Duivenvoorden et al "2012.10437",](https://arxiv.org/abs/2012.10437) [gitlab.mpcdf.mpg.de/mtr/ducc](https://gitlab.mpcdf.mpg.de/mtr/ducc)

To focus on the impact of HWP non-idealities we consider a simplified problem:

- $\triangleright$  no noise.
- $\blacktriangleright$  single frequency,
- $\triangleright$  CMB-only,
- $\blacktriangleright$  simple beams,
- $\blacktriangleright$  HWP aligned to the detector line of sight.

(minimal) TOD: signal detected by 4 detectors.

$$
\begin{pmatrix} d^{(0)} \\ d^{(90)} \\ d^{(45)} \\ d^{(135)} \end{pmatrix} = \begin{pmatrix} (1\ 1\ 0) \cdot \mathcal{R}_{0-\phi} \mathcal{M}_{\text{HWP}} \mathcal{R}_{\phi+\psi} \\ (1\ 1\ 0) \cdot \mathcal{R}_{90-\phi} \mathcal{M}_{\text{HWP}} \mathcal{R}_{\phi+\psi} \\ (1\ 1\ 0) \cdot \mathcal{R}_{45-\phi} \mathcal{M}_{\text{HWP}} \mathcal{R}_{\phi+\psi} \end{pmatrix} \cdot \begin{pmatrix} I_{\text{in}} \\ Q_{\text{in}} \\ U_{\text{in}} \end{pmatrix}
$$



(minimal) TOD: signal detected by 4 detectors.





### modeling the observed maps

map-maker: bin-averaging  $\widehat{S} = (\widehat{A}^T \widehat{A})^{-1} \widehat{A}^T A \cdot S$  assuming ideal HWP.

$$
\widehat{\bm{\mathcal{A}}} = \begin{pmatrix} (1\ 1\ 0) \cdot \mathcal{R}_{\mathbf{0} - \phi} \mathcal{M}_{\text{ideal}} \mathcal{R}_{\phi + \psi} \\ (1\ 1\ 0) \cdot \mathcal{R}_{\mathbf{90} - \phi} \mathcal{M}_{\text{ideal}} \mathcal{R}_{\phi + \psi} \\ (1\ 1\ 0) \cdot \mathcal{R}_{\mathbf{45} - \phi} \mathcal{M}_{\text{ideal}} \mathcal{R}_{\phi + \psi} \\ (1\ 1\ 0) \cdot \mathcal{R}_{\mathbf{135} - \phi} \mathcal{M}_{\text{ideal}} \mathcal{R}_{\phi + \psi} \end{pmatrix}
$$



### estimated ouput maps

$$
\hat{l} = m_{ii} l_{in} + (m_{iq} Q_{in} + m_{iu} U_{in}) \cos(2\alpha) + (m_{iq} U_{in} - m_{iu} Q_{in}) \sin(2\alpha),
$$
  
\n
$$
\hat{Q} = \frac{1}{2} \Big\{ (m_{qq} - m_{uu}) Q_{in} + (m_{qu} + m_{uq}) U_{in} + 2m_{qi} l_{in} \cos(2\alpha) + 2m_{ui} l_{in} \sin(2\alpha) + [(m_{qq} + m_{uu}) Q_{in} + (m_{qu} - m_{uq}) U_{in}] \cos(4\alpha) + [-(m_{qu} - m_{uq}) Q_{in} + (m_{qq} + m_{uu}) U_{in}] \sin(4\alpha) \Big\},
$$
  
\n
$$
\hat{U} = \frac{1}{2} \Big\{ (m_{qq} - m_{uu}) U_{in} - (m_{qu} + m_{uq}) Q_{in} - 2m_{ui} l_{in} \cos(2\alpha) + 2m_{qi} l_{in} \sin(2\alpha) + [-(m_{qq} + m_{uu}) U_{in} + (m_{qu} - m_{uq}) Q_{in}] \cos(4\alpha) + [(m_{qu} - m_{uq}) U_{in} + (m_{qq} + m_{uu}) Q_{in}] \sin(4\alpha) \Big\},
$$

where  $\alpha = \phi + \psi$ . For **good** coverage and **rapidly spinning** HWP:

$$
\widehat{\mathsf{S}}\simeq\begin{pmatrix}m_{ii}l_{\mathsf{in}}\\[(m_{qq}-m_{uu})Q_{\mathsf{in}}+(m_{qu}+m_{uq})U_{\mathsf{in}}]/2\\[(m_{qq}-m_{uu})U_{\mathsf{in}}-(m_{qu}+m_{uq})Q_{\mathsf{in}}]/2\end{pmatrix}.
$$

# Expanding  $\widehat{S}$  in spherical harmonics:

$$
\begin{split}\n\widehat{C}_{\ell}^{TT} &\simeq m_{ii}^{2} C_{\ell,in}^{TT}, \\
\widehat{C}_{\ell}^{EE} &\simeq \frac{(m_{qq} - m_{uu})^{2}}{4} C_{\ell,in}^{EE} + \frac{(m_{qu} + m_{uq})^{2}}{4} C_{\ell,in}^{BB} + \frac{(m_{qq} - m_{uu})(m_{qu} + m_{uq})}{2} C_{\ell,in}^{EB}, \\
\widehat{C}_{\ell}^{BB} &\simeq \frac{(m_{qq} - m_{uu})^{2}}{4} C_{\ell,in}^{BB} + \frac{(m_{qu} + m_{uq})^{2}}{4} C_{\ell,in}^{EE} - \frac{(m_{qq} - m_{uu})(m_{qu} + m_{uq})}{2} C_{\ell,in}^{EB}, \\
\widehat{C}_{\ell}^{TE} &\simeq \frac{m_{ii}(m_{qq} - m_{uu})}{2} C_{\ell,in}^{TE} + \frac{m_{ii}(m_{qu} + m_{uq})}{2} C_{\ell,in}^{TB}, \\
\widehat{C}_{\ell}^{EB} &\simeq \frac{(m_{qq} - m_{uu})^{2} - (m_{qu} + m_{uq})^{2}}{4} C_{\ell,in}^{EB} - \frac{(m_{qq} - m_{uu})(m_{qu} + m_{uq})}{2} (C_{\ell,in}^{EE} - C_{\ell,in}^{BB}), \\
\widehat{C}_{\ell}^{TB} &\simeq \frac{m_{ii}(m_{qq} - m_{uu})}{2} C_{\ell,in}^{TB} - \frac{m_{ii}(m_{qu} + m_{uq})}{2} C_{\ell,in}^{TE}.\n\end{split}
$$

#### analytical vs simulated output spectra



<span id="page-39-0"></span>[a first application:](#page-39-0) [impact on cosmic birefringence](#page-39-0)

# cosmic birefringence in harmonic space

Rotation of the plane of linear polarization of photons travelling through a time dependent pseudoscalar field.



Mixing of  $E$  and  $B$  modes:

$$
\begin{cases}\na_{\ell m, \text{obs}}^E = a_{\ell m}^E \cos 2\beta - a_{\ell m}^B \sin 2\beta, \\
a_{\ell m, \text{obs}}^B = a_{\ell m}^E \sin 2\beta + a_{\ell m}^B \cos 2\beta.\n\end{cases}
$$

image credit: Yuto Minami

### hints of cosmic birefringence

$$
\begin{cases}\nC_{\ell,obs}^{TT} = C_{\ell}^{TT}, \nC_{\ell,obs}^{EE} = \cos^{2}(2\beta)C_{\ell}^{EE} + \sin^{2}(2\beta)C_{\ell}^{BB} - \sin(4\beta)C_{\ell}^{EB}, \nC_{\ell,obs}^{BB} = \cos^{2}(2\beta)C_{\ell}^{BB} + \sin^{2}(2\beta)C_{\ell}^{EE} + \sin(4\beta)C_{\ell}^{EB}, \nC_{\ell,obs}^{TE} = \cos(2\beta)C_{\ell}^{TE} - \sin(2\beta)C_{\ell}^{TB}, \nC_{\ell,obs}^{EB} = \sin(4\beta)(C_{\ell}^{EE} - C_{\ell}^{BB})/2 + \cos(4\beta)C_{\ell}^{EB}, \nC_{\ell,obs}^{TB} = \sin(2\beta)C_{\ell}^{TE} + \cos(2\beta)C_{\ell}^{TB}.\n\end{cases}
$$
\n
$$
\begin{cases}\nC_{\ell,obs}^{EB} = \tan(4\beta)(C_{\ell,obs}^{EE} - C_{\ell,obs}^{BB})/2.\n\end{cases}
$$

### hints of cosmic birefringence

$$
\begin{cases}\nC_{\ell,obs}^{TT} = C_{\ell}^{TT}, \\
C_{\ell,obs}^{EE} = \cos^2(2\beta)C_{\ell}^{EE} + \sin^2(2\beta)C_{\ell}^{BB} - \sin(4\beta)C_{\ell}^{EB}, \\
C_{\ell,obs}^{BB} = \cos^2(2\beta)C_{\ell}^{BB} + \sin^2(2\beta)C_{\ell}^{EE} + \sin(4\beta)C_{\ell}^{EB}, \\
C_{\ell,obs}^{TE} = \cos(2\beta)C_{\ell}^{TE} - \sin(2\beta)C_{\ell}^{TB}, \\
C_{\ell,obs}^{EB} = \sin(4\beta)(C_{\ell}^{EE} - C_{\ell}^{BB})/2 + \cos(4\beta)C_{\ell}^{EB}, \\
C_{\ell,obs}^{TB} = \sin(2\beta)C_{\ell}^{TE} + \cos(2\beta)C_{\ell}^{TB}.\n\end{cases}
$$
\n
$$
\begin{aligned}\nC_{\ell,obs}^{EB} = \tan(4\beta)(C_{\ell,obs}^{EE} - C_{\ell,obs}^{BB})/2, \\
\beta = 0.35 \pm 0.14 (68\% \text{CL}) \\
\text{To be confirmed (or not) by future polarization observations!}\n\end{aligned}
$$

Minami and Komatsu (2020) Phys. Rev. Lett. 125

Analytic  $C_{\ell}$ s satisfy the relations:  $\left(\widehat{\mathcal{C}}^{EB}_\ell \simeq \tan(4\widehat{\theta}) \left[\widehat{\mathcal{C}}^{EE}_\ell - \widehat{\mathcal{C}}^{BB}_\ell \right]/2 \right]$  $\widehat{C}^{TB}_{\ell} \simeq \tan(2\widehat{\theta}) \widehat{C}^{TE}_{\ell}$ 

> The HWP induces an additional miscalibration, degenerate with cosmic birefringence and polarization angle miscalibration!



The HWP induces an additional miscalibration, degenerate with cosmic birefringence and polarization angle miscalibration!

This doesn't mean that the HWP will keep us from measuring  $\beta$ , but it shows how important it is to carefully calibrate  $M_{HWP}$ .

<span id="page-45-0"></span>[work in progress:](#page-45-0) [end-to-end model](#page-45-0)

For the single frequency model, we started from  $d = (1 0 0) \cdot \mathcal{M}_{\text{det}} \mathcal{R}_{\epsilon-\phi} \mathcal{M}_{\text{HWP}} \mathcal{R}_{\phi+\psi} \cdot S$ , which can be generalized to account for the **frequency dependence** of  $M_{\text{HWP}}$  and signal:

$$
d^i = (1\ 0\ 0) \cdot \mathcal{M}_{\mathsf{det}} \mathcal{R}_{\xi-\phi} \int_{\nu_{\mathsf{min}}^i}^{\nu_{\mathsf{max}}^i} \frac{\mathrm{d} \nu}{\Delta \nu^i} \mathcal{M}_{\mathsf{HWP}}(\nu) \mathcal{R}_{\phi+\psi} \cdot \mathsf{S}^i(\nu) \,.
$$

Additional generalizations (more components, beams and noise) are also easy to implement, and the resulting model can be used as a starting point to retrace the same steps as before.

#### modeling the multi-frequency maps

With HWP: 
$$
\hat{m}^i \simeq \sum_{\lambda} \begin{pmatrix} g_{\lambda}^i & 0 & 0 \\ 0 & \rho_{\lambda}^i & \eta_{\lambda}^i \\ 0 & -\eta_{\lambda}^i & \rho_{\lambda}^i \end{pmatrix} \overline{m}_{\lambda}^i(\nu) + n^i
$$
,  
\nwhere  $g_{\lambda}^i \equiv \int_{\nu_{\min}^i}^{\nu_{\max}^i} \frac{d\nu}{\Delta \nu^i} a_{\lambda}(\nu) m_{ii}(\nu)$ ,  
\n $\rho_{\lambda}^i \equiv \frac{1}{2} \int_{\nu_{\min}^i}^{\nu_{\max}^i} \frac{d\nu}{\Delta \nu^i} a_{\lambda}(\nu) [m_{qq}(\nu) - m_{uu}(\nu)]$ ,  
\n $\eta_{\lambda}^i \equiv \frac{1}{2} \int_{\nu_{\min}^i}^{\nu_{\max}^i} \frac{d\nu}{\Delta \nu^i} a_{\lambda}(\nu) [m_{qu}(\nu) + m_{uq}(\nu)]$ .

How the HWP non-idealities affect gain, polarization-efficiency and cross-pol leakage, differ for each frequency channel and each component.

# moving forward

$$
\boxed{\{\widehat{\mathsf{m}}^i,\ldots,\widehat{\mathsf{m}}^\mathit{n_\text{chan}}\}}\quad\text{with}\quad\widehat{\mathsf{m}}^i\simeq\sum_{\lambda}\begin{pmatrix}g^i_\lambda&0&0\\0&\rho^i_\lambda&\eta^i_\lambda\\0&-\eta^i_\lambda&\rho^i_\lambda\end{pmatrix}\,\overline{\mathsf{m}}^{\,i}_\lambda(\nu)+\mathsf{n}^i\,.
$$

# moving forward

$$
\frac{\{\widehat{\mathsf{m}}^i, \ldots, \widehat{\mathsf{m}}^{n_{\text{chan}}}\}}{\begin{array}{c}\text{parametric or blind component separation}\\ \text{[HILC output can be modeled analytically)}\end{array}} \begin{pmatrix} g^i_\lambda & 0 & 0\\ 0 & \rho^i_\lambda & \eta^i_\lambda\\ 0 & -\eta^i_\lambda & \rho^i_\lambda \end{pmatrix}} \overline{\mathsf{m}}^i_\lambda(\nu) + \mathsf{n}^i \,.
$$

# moving forward

$$
\frac{\{\widehat{\mathsf{m}}^i, \ldots, \widehat{\mathsf{m}}^{n_{\text{chan}}}\}}{\begin{array}{c}\text{parameter of blind component separation}\\\text{left control in the model,}\\ \text{int of the model,}\\ \text{int of the model}\end{array}} \begin{array}{c}\begin{pmatrix}g^i_\lambda & 0 & 0\\ 0 & \rho^i_\lambda & \eta^i_\lambda\\ 0 & -\eta^i_\lambda & \rho^i_\lambda\end{pmatrix}} \end{array} \begin{array}{c}\overline{\mathsf{m}}^i_\lambda(\nu) + \mathsf{n}^i \cdot \\\begin{pmatrix}g^i_\lambda & 0 & 0\\ 0 & -\eta^i_\lambda & \rho^i_\lambda\end{pmatrix}} \end{array}
$$
\nFIG-cleaned CMB map.

\nlikelihood maximization

\nEstimates of *r* and/or  $\beta$ .

### conclusions and outlook

- $\triangleright$  Much information is still hidden in CMB polarization, for instance primordial B-modes and cosmic birefringence as probes of, respectively, inflationary and parity-violating physics,
- $\triangleright$  New physics can be probed only if systematics are well under control,
- $\triangleright$  A rotating HWP can help, but it induces additional systematics which should be accounted for (HWP-induced miscalibration),
- $\triangleright$  We are now provided with an analytical model and a simulation pipeline that can be used to study the impact of the HWP in more realistic scenarios. this is key for the planning of the next generation of CMB experiments.

# <span id="page-52-0"></span>**[backup](#page-52-0)**

# sketch of the pipeline

beamconv: convolution code simulating TOD for CMB experiments with realistic polarized beams, scanning strategies and HWP.

DUCC: collection of basic programming tools for numerical computation: fft, sht, healpix, totalconvolve...



[github.com/AdriJD/beamconv,](https://github.com/AdriJD/beamconv) [A. Duivenvoorden et al "2012.10437",](https://arxiv.org/abs/2012.10437) [gitlab.mpcdf.mpg.de/mtr/ducc](https://gitlab.mpcdf.mpg.de/mtr/ducc)

### input maps

![](_page_54_Figure_1.jpeg)

The pipeline can be fed with arbitrary input maps: CMB, foregrounds, or both.

In the paper:  $I$ ,  $Q$  and  $U$  input maps with  $n_{\text{side}} = 512$  from best-fit 2018 Planck power spectra;

#### scanning strategy

![](_page_55_Figure_1.jpeg)

The pipeline can read or calculate point-

[https://github.com/tmatsumu/LB\\_SYSPL\\_v4.2](https://github.com/tmatsumu/LB_SYSPL_v4.2)

# focal plane specifics

![](_page_56_Figure_1.jpeg)

The pipeline can read from the Instrument Model Database (IMO):

```
{'name': 'M02_030_QA_140T',
 'wafer': 'M02',
 'pixel': 30,
 'pixtype': 'MP1',
 [...]
 'pol': 'T',
 'orient': 'Q',
 'quat': [1, 0, 0, 0]}
```
In the paper: 160 dets from M1-140.

![](_page_56_Picture_144.jpeg)

![](_page_57_Figure_1.jpeg)

In the paper: HWP is assumed to be ideal in the first simulation run (ideal TOD) and realistic in the second (nonideal TOD).

Realistic HWP Mueller matrix elements as shown previously:

![](_page_57_Figure_4.jpeg)

![](_page_58_Figure_1.jpeg)

![](_page_58_Picture_2.jpeg)

![](_page_59_Figure_1.jpeg)

- $\blacktriangleright$  TT leaked a bit
- $\blacktriangleright$  *FE* leaked a lot!

![](_page_60_Figure_1.jpeg)

- $\blacktriangleright$  TT leaked a bit
- $\blacktriangleright$  *FE* leaked a lot!
- $\triangleright$  BB larger (EE shape!)

![](_page_61_Figure_1.jpeg)

- $\blacktriangleright$  TT leaked a bit
- $\blacktriangleright$  *FE* leaked a lot!
- $\triangleright$  BB larger (EE shape!)
- $\blacktriangleright$  TE leaked a bit

![](_page_62_Figure_1.jpeg)

- $\blacktriangleright$  TT leaked a bit
- $\blacktriangleright$  *FE* leaked a lot!
- $\triangleright$  BB larger (EE shape!)
- $\blacktriangleright$  TE leaked a bit
- $\blacktriangleright$  EB non-zero!

![](_page_63_Figure_1.jpeg)

- $\blacktriangleright$  TT leaked a bit
- $\blacktriangleright$  *FE* leaked a lot!
- $\triangleright$  BB larger (EE shape!)
- $\blacktriangleright$  TE leaked a bit
- $\blacktriangleright$  EB non-zero!
- TB non-zero!

![](_page_64_Figure_1.jpeg)

Birefringence: property of a material whose refractive index depends on the polarization and propagation direction of light.

![](_page_65_Picture_2.jpeg)

Thinner slabs, normal incidence: no double refraction, only retardance.

![](_page_65_Figure_4.jpeg)

Both optical and cosmic birefringence rotate polarization vectors.

# instrument miscalibration

![](_page_66_Figure_1.jpeg)

Taking such (frequency-independent) deviations into account:

$$
\widehat{\theta} = -\frac{1}{2} \arctan \frac{\langle m_{qu} + m_{uq} \rangle}{\langle m_{qq} - m_{uu} \rangle} + \delta \theta,
$$

where  $\delta\theta \equiv \delta\xi - \delta\psi - 2\delta\phi$ .